OFFICE OF NAVAL RESEARCH

FINAL REPORT

January 1, 1989 through December 31, 1991

for
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"MATHEMATICAL METHODS IN OPERATIONS RESEARCH
AND COMPUTER SCIENCE"



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**OFFICE OF NAVAL RESEARCH**
Final Report

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1. Our progress on the funoamental problem of decision science, solving large-scale optimization problems whose parameters are subject to uncertainty, has been so rapid during the past year that we are now in a position to tackle the most general type of stochastic models one is likely to encounter in practice. It is particularly relevant to the Navy during this critical period of deciding how best to down-size the military and yet maintain a force robust and ready to handle any combination of contingencies that might arise in the future with high probability.

2. A major breakthrough in this area has the potential of profoundly affecting the quality of planning, the reduction of cycle time to improve products, the reduction of time to militarily deploy in crises, and generally improve the industrial competitiveness of U.S. until the time that other countries catch on.

3. By way of background, our approach uses a combination of decomposition methods (D-W and Benders), importance sampling, and (if necessary) parallel processors. Our advances on this very complex topic are not solely the result of good theoretical ideas, but are also due to the special skills and drive to succeed on the part of our little research team. By extraordinary good luck, Dr. Gerd Infanger, on leave from the Technical University of Vienna, joined us in early 1989. Trained as an electrical engineer, his techniques are used to schedule the distribution of electric power in Austria. Gerd's theoretical ideas, his enthusiasm, and his skill as a software implementor have transformed our research progress over night.

4. We began tests a year and a half ago on a relevant broad class of multi period models that could be treated as essentially two-staged because the effects of uncertainties in various periods were independent from one another. When it turned out we could successfully handle this class, we moved next to a fairly general class of dynamic models where the matrix structure of activities (the technology) within a period were known with certainty but their outputs (which were the inputs to the next period) were not known with certainty nor were their exogenous inputs known with certainty.

From a practical point of view probably the most general models that one would ever want to look at in practice are those where there is a correlation between periods or stages (such as correlation between the uncertain demands for electricity or water/flow from one period to the next). Another type of correlation is uncertainty about a possible improvement in technology; if the improvement does become available in one period, it will certainly be available in the next. We believe we now know how to handle this very general class, and this is what our research concentrated on last year.

5. In the last five years there has been a major development in the technology of computer architecture with the promise that it may be possible in the near future to buy fairly cheaply computers with many parallel and vector processors, each having the power of main frames. It is this that inspired us to seriously consider solving stochastic programs, for the major bottle neck (historically) has been that **stochastic** programs when expressed as equivalent **deterministic** programs can be truly enormous -- billions of constraints for example. Typically their submatrices are interrelated in a branching structure with many branches in parallel.

This gave rise to the fascinating possibility of not actually trying to scan inside the computer each member of the universe of possible contingencies but to find a way to **sample** this universe (just as it is not necessary to measure the height of all the people in the U.S. to get a guaranteed close estimate of the true average height of people; a relatively small random sample will do). The fascinating possibility was to use Monte Carlo sampling. With the possibility of parallel processors in the offing, it took no imagination to see how each of the processors of a parallel processor could be put to work in parallel with the others busy carrying out the sampling. However, Professor Peter Glynn, an expert on Importance Sampling and a member of our research team, pointed out that while Monte Carlo sampling may look like a good idea, the sample size to get an accurate answer will likely turn out to be too large to be practical when we employ what is called "naive" sampling procedures.

Glynn's advice was to employ instead **importance sampling** in order to keep the size of sample down. We have discovered an **adaptive procedure** for generating the approximating function used in the method. This discovery has so reduced the size of sample needed to get accurate results that in most cases we can solve hitherto unsolvable practical stochastic problems without the need of parallel or vector processors, indeed, they turned out to be solvable on PCs with fast chips. We have successfully solved realistic test models drawn from electric-power industry and from financial investment, with millions of contingencies (events) whose deterministic linear programming equivalent would have had over a billion constraints.

6. We believe by means of Benders Decomposition and Importance Sampling, we have achieved a breakthrough. A breakthrough, however, is not to be interpreted to mean the same as realization. Much hard work and funding to provide continuity of support of the research team is necessary to bridge the gap between the theoretical advance and its practical realization. Hard work on the part of others developing suitable applications is also necessary.

7. The above concentrates on our progress on this fundamental problem of decision science, the research of Richard Cottle and those associated with him concentrated on linear complementarity. Recently R.W. Cottle, J.S. Pang and R.E. Stone completed a monumental book on the subject entitled *The Linear Complementarity Problem*, Academic Press.

8. Below is a listing of Technical Report Publications partially supported by ONR, abstracts of these reports, and books published.

**TECHNICAL REPORT PUBLICATIONS:**

SOL 89-1 **HU, Hui.** On the Feasibility of a Generalized Linear Program, January 1989.

SOL 89-2 **HU, Hui.** Semi-Infinite Programming, March 1989. **Mathematical Programming**.

SOL 89-3 **COTTLE, Richard W.** The Principal Pivoting Method Revisited, April 1989. Published in **Mathematical Programming 48**, Series B, (1990) North-Holland, pp. 369-385.

SOL 89-4 **KRISHNA, Alamuru.** Note on Degeneracy, April 1989. Submitted for publication in **Mathematical Programming**.

SOL 89-5 **ZIKAN, Karel.** An Efficient Exact Algorithm for the "LEAST-SQUARES" Image Registration Problem, May 1989. **Pattern Analysis and Machine Intelligence**.

**SOL 89-10** **de MAZANCOURT, Pierre F.** (Ph.D. thesis) A Matrix Factorization And Its Application To Large-Scale Linear Programming, July 1989.

**SOL 89-13** **INFANGER, Gerd.** Monte Carlo (Importance) Sampling within a Benders' Decomposition Algorithm for Stochastic Linear Programs, September 1989. **Revised August 1990**. Accepted for publication in the special issue of **Annals of Operations Research**.

**SOL 89-15** **YAO, Jen-Chih.** Generalized Quasi-Variational Inequality and Implicit Complementarity Problems, October 1989. "The Generalized Implicity Complementarity Problem with Applications" to appear in **Journal of Mathematical Analysis and Applications**. Also submitted to **Journal of Mathematical Analysis and Applications**.

**SOL 89-16** **YAO, Jen-Chih.** A Basic Theorem of Complementarity for the Generalized Variational-like Inequality Problem, November 1989. To appear in **Journal of Mathematical Analysis and Applications**.

**SOL 89-17** **ENTRIKEN, Robert E.** (Ph.D. thesis) The Parallel Decomposition of Linear Programs, November 1989. Chapter 4 submitted to **ORSA Journal on Computing**.

**SOL 89-18** **YAO, Jen-Chih.** On Mean Value Iterations with Application to Variational Inequality Problems, December 1989.

**SOL 89-19** **YAO, Jen-Chih.** Fixed Points by Ishikawa Iterations, December 1989. Submitted to **Journal of Mathematical Analysis and Applications**.

**SOL 90-4** **DANTZIG, George B. and Yinyu Ye.** A Build-up Interior Method for Linear Programming, February 1990. To appear in **Mathematical Programming**.

**SOL 90-5** **YAO, Jen-Chih.** A Generalized Complementarity Problem in Hilbert Space, March 1990. Submitted to **Journal of Mathematical Analysis and Application**.

**SOL 90-7** **YAO, Jen-Chih.** Monotone Complementarity Problem in Hilbert Space, April 1990. Submitted to **Bulletin of the Australian Mathematical Society**.

**SOL 90-9** **COTTLE, Richard W. and Yow-Yieh Chang.** Least-Index Resolution of Degeneracy in Linear Complementarity Problems with Sufficient Matrices, June 1990. To Appear in **SIAM Journal of Matrix Analysis and Application**.

**SOL 90-10** **COTTLE, Richard W. and Jen-Chih YAO.** Pseudo-monotone Complementarity Problems in Hilbert Space, July 1990. **Journal of Optimization Theory and Applications**, Vol. 75, 2, November 1992.

**SOL 90-11** **SCHWEITZER, Eithan.** Modifying MINOS for Solving the Dual of a Linear Program, August 1990. Submitted to **Mathematical Programming**.

**SOL 90-17** **COTTLE, Richard W. and Sy-Ming GUU.** Two Characterizations of Sufficient Matrices, December 1990. **Linear Algebra and Its Applications**, Vol. 170, pp. 65-74, 1992.

**SOL 91-4** **DANTZIG, George B. and Gerd INFANGER.** Large-Scale Stochastic Linear Programs: Importance Sampling and Benders Decomposition, March 1991. Submitted for publication in **Mathematical Programming**.

SOL 91-5 **DANTZIG, George B.** Convering a Converging Algorithm into a Polynomially Bounded Algorithm, March 1991.

SOL 91-6 **INFANGER, Gerd.** Monte Carlo (Importance) Sampling within a Benders Decomposition Algorithm for Stochastic Linear Programs Extended Version: Including Results of Large-Scale Problems, March 1991. Accepted for publication in the special issue of **Annals of Operations Research**.

SOL 91-10 **DANTZIG, George, B., James K. HO and Gerd INFANGER.** Solving Stochastic Linear Programs on a Hypercube Multicomputer, August 1991. Submitted for publication in **Operations Research**.

SOL 91-11 **DANTZIG, George B. and Gerd INFANGER.** Multi-Stage Stochastic Linear Programs for Portfolio Optimization, September 1991. To be published at the **Proceedings of the Annual Symposium of RAMP** (Research Association of Mathematical Programming) in Tokyo, November 1991.

SOL 92-1 **LEICHNER, S. A., G. B. DANTZIG and J. W. DAVIS.** A Strictly Improving Phase I Algorithm Using Lease-Squares Subproblems, April 1992.

SOL 92-2 **LEICHNER , S. A., G. B. DANTZIG and J. W. DAVIS.** A Strictly Improving Linear Programming Algorithm Based on a Series of Phase I Problems, April 1992.

**TECHNICAL REPORT ABSTRACTS:**

**SOL 89-1: On the Feasbility of a Generalized Linear Program, Hui Hu (January 1989, 6 pp.).**

The first algorithm for solving generalized linear programs was given by George B.Dantzig. His algorithm assumes that a basic feasible solution of the generalized linear program to be solved exists and is given. If the initial basic feasible solution is non-degenerate, then his algorithm is guaranteed to converge. The purpose of this paper is to show how to find an initial basic feasible (possibly degenerate) solution of a generealized linear program by applying the same algorithm to a "phase-one" problem without requiring that the initial basic feasible solution to the latter be non-degenerate.

**SOL 89-2: Semi-Infinite Programming, Hui Hu (March 1989, 59 pp.).**

Semi-Infinite programming, that allows for either infinitely many constraints or infinitely many variables but not both, is a natural extension of ordinary mathematical programming. There are many practical as well as theoretical problems in which the constraints depend on time or space and thus can be formulated as semi-infinite programs. The focus of this dissertation is on formulating and solving semi-infinite programming problems. The main results include:

(1) An algorithm for solving a matrix rescaling problem formulated as a semi-infinite linear program. Sufficient conditions that guarantee finite termination are discussed and computational results are reported.

(2) An algorithm for solving a matrix estimation problem equivalent to a semi-infinite quadratic program. For a specified constant, this algorithm will find an approximate solution after finitely many iterations, or will tend to an optimal solution in the limit. An upper bound on the total number of iterations needed for finding an approximate solution is given. Computational results are reported.

(3) A one-phase algorithm for solving a large class of semi-infinite linear programming problems. This algorithm has several advantages: it handles feasibility and optimality together and can detect infeasibility after a finite number of iterations; it has very weak restrictions on the constraints; it allows cuts that are not near the most violated cut; and it solves the primal and the dual problems simultaneously. Upper bounds for finding an $\epsilon$-optimal solution and for the distance between an $\epsilon$-optimal solution and an optimal solution are given.

(4) Applications of the above algorithm to convex programming. First, a certain semi-infinite linear program is solved by this algorithm so as to obtain a feasible solution of a convex program. Then, another semi-infinite linear program is solved by this algorithm so as to obtain an optimal solution of the convex program. In particular, it is shown that for a strongly consistent convex program this algorithm can find a feasible solution after a finite number of iterations.

**SOL 89-3: The Principal Pivoting Method Revisited, Richard W. Cottle, (April 1989, 18 pp.).**

The Principal Pivoting Method (PPM) for the Linear Complementarity Problem (LCP) is shown to be applicable to the class of LCPs involving the newly identified class of sufficient matrices.

**SOL 89-4: Note on Degeneracy, Alamuru S. Krishna (April 1989, 3 pp.).**

Given a linear program in standard form, Min $cx$ $s.t.$ $Ax = b$, $x \geq 0$, where A is an $m \times n$ matrix with rational coefficients, one technique used to resolve degeneracy in the simplex algorithm is the lexicographic rule. This rule adjoins to $b$ a non-singular $m \times m$ square matrix $M$. The appended columns of $M$ are updated along with $b$ on each iteration. When the ratio test for determining the pivot row results in a tie, the ratio test is applied to the corresponding elements of the updated columns of $M$ in turn, from left to right, until the tie is resolved. In this note we prove that it is only necessary instead, to adjoin to $b$ a single column $d$ whose ith component is $d_i = \pi^i$, ( or preferably the fractional part of $\pi^i$ in order that $|d_i| < 1$). Any transcendental number, like $e = 2.73\ldots$, the base of the natural logirthm, can be used instead of $\pi = 3.14\ldots$. The proof exploits the fundamental property of a transcendental number namely, it can never be a root of a polynomial equation $\alpha_1 x + \alpha_2 x^2 + \cdots + \alpha_p x^p = 0$ when $\alpha_1, \alpha_2, \ldots, \alpha_p$ are rational and not all zero.

**SOL 89-5: An Efficient Exact Algorithm for the "LEAST SQUARES" Image Registration Problem, Karel Zikan (May 1989, 14 pp.).**

Image registration involves estimating how one set of $n$-dimensional points is rotated, scaled, and translated into a second set of $n$-dimensional points. In practice, $n$ is usually 2 or 3. We give an exact algorithm to solve the "least squares" formulation of the two-dimensional registration problem. The algorithm, which is based on parametric linear programming, can be viewed as a refinement of the $O(k^3)$ approximation method proposed by Zikan and Silberberg [13]. The approach can be extended to handle registration of images of difference cardinalities.

**SOL 89-10: A Matrix Factorization And Its Application to Large-Scale Linear Programming, Pierre F. de Mazancourt (July 1989, 81 pp.).**

As an alternative to the LU matrix factorization, we consider a factorization that uses the lower triangular part of the original matrix as one factor and computes the other factors as a product of rank-one update matrices:

Under some non-singularity assumptions, an $m \times m$ matrix $A$ can be factorized as $E_m E_{m-1} \ldots E_2 A_1$ where $A_1$ is the lower triangular part of $A$ and $E_k$ is a rank-one update matrix of the form $I + v_k w_k$ with $v_k$ a column vector and $w_k$ a row vector. The vector $v_k$ is the $k^{th}$ column of $A - A_1$. If $v_k = ze$, then $E_k = I$ may be omitted from the factorization. Otherwise, the row vector $w_k$ must be computed.

After reviewing and improving the time complexity, the requirements, the stability and the efficiency of this method, we derive a stable factorization algorithm which we implement in FORTRAN77 within the framework of the simplex algorithm for linear programming.

A comparison of our numerical results with those obtained through the code MINOS 5.3 indicate that our method may be more efficient than an ordinary LU decomposition for some matrices whose order ranges between 28 and 1481, especially when these matrices are almost triangular.

**SOL 89-13: Monte Carlo (Importance) Sampling within a Benders' Decomposition Algorithm for Stochastic Linear Programs, Gerd Infanger (September 1989, 30 pp.).**

A method employing decomposition techniques and Monte Carlo sampling (importance sampling) to solve stochastic linear programs is described and applied to capacity expansion planning problems of electric utilities. We consider uncertain availability of generators and transmission lines and uncertain demand. Numerical results are presented.

**SOL 89-15: Generalized Quasi-Variational Inequality and Implicit Complementarity Problems, Jen-Chih Yao (October 1989, 47 pp.).**

A new problem called the *generalized quasi-variational inequality problem* is introduced. This new formulation extends all kinds of variational inequality problem formulations that have been introduced and enlarges the class of problems that can be approached by the variational inequality problem formulation. Existence results without convexity assumptions are established and topological properties of the solution set are investigated. A new problem call the *generalized implicit complementarity problem* is also introduced which generalizes all the complementarity problem formulations that have been introduced. Applications of generalized quasi-variational inequality and implicit complementarity problems are given.

**SOL 89-16: A Basic Theorem of Complementarity for the Generalized Variational-like Inequality Problem, Jen-Chih Yao (November 1989, 17 pp.).**

In this report, a basic theorem of complementarity is established for the generalized variational-like inequality problem introduced by Parida and Sen. Some existence results for both generalized variational inequality and complementarity problems are established by employing this basic theorem of complementarity. In particular, some sets of conditions that are normally satisfied by a nonsolvable generalized complementarity problem are investigated.

**SOL 89-17: The Parallel Decomposition of Linear Programs, Robert Entriken (November 1989, 150 pp.).**

This thesis introduces a new calculus for manipulating linear-program decomposition schemes. A linear program is represented by a *communication network*, which is decomposed by splitting nodes in two, and a transformation is defined to recover subproblems from the network. We also define a dual-symmetric oracle that provides solutions to linear programs, and can be performed by the simplex method, nested decomposition, and finally, parallel decomposition.

Two important classes of linear program serve as examples for the above calculus: staircase linear programs and stochastic linear programs. For the former case, a sophisticated yet experimental computer code has been written for an IBM 3090/600E with six processors. The code performs the parallel decomposition algorithm and is tested on twenty-two small to medium sized "real-world" problems. Experiments show that in addition to speedups provided by decomposition alone, performance is improved by using parallel processors.

**SOL 89-18: On Mean Value Iterations with Application to Variational Inequality Problems, Jen-Chih Yao (December 1989, 8 pp.).**

In this report, we show that in a Hilbert space, a mean value iterative process generated by a continuous quasi-nonexpansive mapping always converges to a fixed point of the mapping without any precondition. We then employ this result to obtain approximating solutions to the variational inequality and the generalized complementarity problems.

**SOL 89-19: Fixed Points by Ishikawa Iterations, Jen-Chih Yao (December 1989, 4 pp.).**

In this paper, we introduce a class of mappings called generalized quasi-nonexpansive mappings in a Hilbert space. It is shown that a certain Ishikawa iterative process generated by a continuous generalized quasi-nonexpansive and monotone mappings on a compact and convex subset of a Hilbert space always converges strongly to a fixed point of the mapping without any precondition.

**SOL 90-4: A Build-Up Interior Method for Linear Programming: Affine Scaling Form, George B. Dantzig and Yinyu Ye (February 1990, 29 pp.).**

We propose a *build-up* interior method for solving an $m$ equation $n$ variable linear program which has the same convergence properties as their well known analogues in dual affine and projective forms but requires less computational effort. The algorithm has three forms, an *affine scaling* form, a *projective scaling* form, and an exact form (that uses pivot steps). In this paper, we present the first of these. It differs from Dikin's algorithm of dual affine form in that the ellipsoid chosen to generate the improving direction $\bar{\Delta}$ in dual space is constructed from only a subset of the dual constraints.

At the start of each major iteration $t$, we are given an interior iterate $y^t$. A selection of $m$ dual constraints is made using an "order-columns" rule as to which constraints show "the most promise" of being tight in the optimal dual solution. An ellipsoid centered at $y^t$ is then inscribed in convex region defined by these promising constraints and an improving direction $\bar{\Delta}$ computed that points to the optimal point $y^t + \bar{\Delta}$ on the ellipsoid boundary. Minor cycling within a major iteration is then started.

During a minor cycle, the constraints selected to define the ellipsoid centered at $y^t$ is built up to include the constraint (whenever there is one) that first blocks feasible movement from $y^t$ to $y^t + \bar{\Delta}$. If one blocks, it is used to augment the set of promising constraints and the ellipsoid is revised; the improving direction $\bar{\Delta}$ is recomputed by means of a rank-one update, and the minor cycle repeated until none blocks movement from $y^t$ to $y^t + \bar{\Delta}$. When none blocks, the minor cycling ends. $y^{t+1} = y^t + \bar{\Delta}$ initiates the next major iteration. Major iterations stop when an optimum solution is reached. We prove this will occur in a finite number of iterations.

**SOL 90-5: A Generalized Complementarity Problem in Hilbert Space, Jen-Chih Yao (March 1990, 5 pp.).**

An existence theorem for a generalized complementarity problem over arbitrary closed convex cone in a Hilbert space is proved.

**SOL 90-7: A Monotone Complementarity Problem in Hilbert Space, Jen-Chih Yao (April 1990, 5 pp.).**

An existence theorem for a complementarity problem involving a weakly coercive monotone mapping over an arbitrary closed convex cone in a real Hilbert space is established.

**SOL 90-9: Least-Index Resolution of Degeneracy in Linear Complementarity Problems with Sufficient Matrices, Richard W. Cottle and Yow-Yieh Chang (June 1990, 14 pp.).**

This paper deals with the Principal Pivoting Method (PPM) for the Linear Complementarity problem (LCP). It is shown here that when the matrix $M$ of the LCP $(q, M)$ is (row and column) sufficient, the incorporation of a least-index pivot selection rule in the PPM makes it a finite algorithm even when the LCP is degenerate.

**SOL 90-10: Pseudo-monotone Complementarity Problems in Hilbert Space, Richard W. Cottle and Jen-Chih Yao (July 1990, 17 pp.).**

In this paper, some existence results for a nonliner complementarity problem involving a pseudo-monotone mapping over an arbitrary closed convex cone in a real Hilbert space are established. In particular, some known existence results for a nonlinear complementarity problem in a finite-dimensional Hilbert speace are generalized to an infinite-dimensional real Hilbert space. Applications to a class of nonlinear complementarity problems and the study of the post-critical equilibrium state of a thin elastic plate subjected to unilateral conditions are given. Submitted to **Mathematical Programming**.

**SOL 90-11: Modifying MINOS for Solving the Dual of a Linear Program, Eithan Schweitzer (August 1990, 48 pp.).**

In solving large-scale linear programs by Benders' decomposition, it can be advantageous to solve the master and the sub problems via their dual problems. In this report I describe the changes I have made in one of the MINOS files, so MINOS could transform a given primal linear program to its dual, solve the dual and in addition, write an MPS file that contains the dual problem.

**SOL 90-17: Two Characterizations of Sufficient Matrices, Richard W. Cottle and Sy-Ming Guu (December 1990, 8 pp.).**

Two characterizations are given for the class of sufficient matrices defined by Cottle, Pang and Venkateswaran. The first is a direct translation of the definition into linear programming terms. The second can be thought of as a generalization of a theorem of T.D. Parsons on P-matrices.

**SOL 91-4: Large-Scale Stochastic Programs: Importance Sampling and Benders Decomposition, George B. Dantzig and Gerd Infanger (March 1991, 14 pp.).**

The paper demonstrates how large-scale stochastic linear programs with recourse can be efficiently solved by using a blending of classical Benders decomposition with a relatively new technique called importance sampling. Numerical results of large-scale problems in the area of expansion planning of power systems and financial planning are presented.

**SOL 91-5:** Converting a Converging Algorithm into a Polynomially Bounded Algorithm, George B. Dantzig (March 1991, 7 pp.).

We consider the general Phase I linear programming problem with a convexity constraint which can be written after some algebraic manipulation in the form:

$$\text{Find } x_j \geq 0, \quad \sum_1^n P_j x_j = 0, \quad \sum_1^n x_j = 1,$$

where $P_j$ are $m$-vectors satisfying $||P_j|| = 1$. If feasible, von Newmann's Center of Gravity Algorithm generates a sequence $t = 1, 2, \ldots$ of approximate solutions $\sum P_j x_j^t = b^t$, $\sum x_j^t = 1$, $x_j^t \geq 0$ which converges in the limit as $t \rightarrow \infty$ to a feasible solution to the Phase I problem. We assume that all perturbed problems $\sum_1^n P_j x_j = \hat{b}$, $\sum x_j = 1$, $x_j \geq 0$ are feasible for all $\| \hat{b} \| < r$ where $r > 0$ is given. We apply this algorithm to $m+1$ perturbed problems with right hand sides $\hat{b} = \hat{b}^i$, $i = 1, 2, \ldots, m+1$ to obtain an exact solution to the unperturbed problem with $\hat{b} = 0$ in $T < 4r^{-2}(m+1)^3$ iterations. Each iteration consists of $m(n+3)\delta$ multiplications and additions where $\delta$ is the non-zero coefficient density.

**SOL 91-6: Monte Carlo (Importance) Sampling within a Benders Decomposition Algorithm for Stochastic Linear Programs Extended Version: Including Results of Large-Scale Problems, Gerd Infanger (March 1991, 39 pp.).**

The paper focuses on Benders decomposition techniques and Monte Carlo sampling (importance sampling) for solving two-stage stochastic linear programs with recourse, a method first introduced by George B. Dantzig and Peter Glynn (1990). The algorithm is discussed and further developed. The paper gives a complete presentation of the method as it is currently implemented. Numerical results from test problems of different areas are presented. Using small test problems we compare the solutions obtained by the algorithm with the universe solutions. We present the solution of large-sclae problems with numerous stochastic parameters which in the deterministic equivalent formulation would have billions of constraints. The problem concern expansion planning of electric utilities with uncertainty in the availabilities of generators and transmission lines and portfolio management with uncertainty in the future returns.

**SOL 91-10: Solving Stochastic Linear Programs on a Hypercube Multicomputer, George B. Dantzig, James K. Ho and Gerd Infanger (August 1991, 34 pp.).**

Large-scale stochastic linear programs can be efficiently solved by using a blending of classical Benders decomposition and a relatively new technique called importance sampling. The paper demonstrates how such an approach can be effectively implemented on a parallel (Hypercube) multicomputer. Numerical results are presented.

**SOL 91-11: Multi-Stage Stochastic Linear Programs for Portfolio Optimization, George B. Dantzig and Gerd Infanger (September 1991, 21 pp.).**

The paper demonstrates how multi-period portfolio optimization problems can be efficiently solved as multi-stage stochastic linear programs. A scheme based on a blending of classical Benders decomposition techniques and a special technique, called importance sampling, is used to solve this general

class of multi-stage stochastic linear programs. We discuss the case where stochastic parameters are deoendent within a period as well as between periods. Initial computational results are presented.

**SOL 92-1: A Strickly Improving Phase I Algorithm Using Least-Squares Subproblems, S. A. Leichner, G. B. Dantzig and J. W. Davis (April 1992, 43 pp.).**

Although the simplex method's performance in solving linear programming problems is usually quite good, it does not guarantee strict improvement at each iteration on degenerate problems. Instead of trying to recognize and avoid degenerate steps in the simplex method (as some variants do), we have developed a new Phase I algorithm that is completely impervious to degeneracy, with a strict improvement attained at each iteration. It is also noted that the new Phase I Algorithm is closely related to a number of existing algorithms.

When tested on the 30 smallest *NETLIB* linear prograggming test problems, the computational results for the new Phase I algorithm were almost 3.5 times faster than the simplex method; on some problems, it was over 10 times faster.

**SOL 92-2: A Strickly Improving Linear Programming Algorithm Based on a Series of Phase I Problems, S. A. Leichner, G. B. Dantzig and J. W. Davis (April 1992, 20 pp.).**

When used on degenerate problems, the simplex method often takes a number of degenerate steps at a particular vertex before moving to the next. In theory (although rarely in practice), the simplex method can actually cycle at such a degenerate point. Instead of trying to modify the simplex method to avoid degenerate steps, we have developed a new linear programming algorithm that is completely impervious to degeneracy.

This new method solves the Phase II problem of finding an optimal solution by solving a series of Phase I feasibility problems. Strict improvement is attained at each iteration in the Phase I algorithm, and the Phase II sequence of feasibility problems has linear convergence in the number of Phase I problems.

When tested on the 30 smallest *NETLIB* linear programming test problems, the computational results for the new Phase II algorithm (using the series of feasiblity problems) were over 15problems, it was almost two times faster, and on one problem it was four times faster.

**BOOKS PUBLISHED:**

Cottle, Richard W., Jong-Shi Pang and Richard E. Stone, *The Linear Complementarity Problem,* Academic Press, Inc., New York, 1992.

Hillier, Frederick S., and Gerald J. Lieberman, *Introduction to Operations Research,* 5th ed., McGraw-Hill, New York, 1990.

Hillier, Frederick S., and Gerald J. Lieberman, *Introduction to Mathematical Programming,* McGraw-Hill, New York, 1990.

Hillier, Frederick S., and Gerald J. Lieberman, *Introduction to Stochastic Models in Operations Research,* McGraw-Hill, New York, 1990.

**INVITED PRESENTATIONS**

*George B. Dantzig*

Invited speaker, Princeton University, November, 1989

Plenary speaker, Financial Optimization Conference, Wharton School; "Developments in Programming Under Uncertainty", November, 1989

Gibbs Lecture, American Mathematical Society, "The Wide Wide World of Pure Mathematics that Goes by Other Names", January , 1990

Invited speaker, Systems and Industrial Engineering Seminar Series, University of Arizona, "Developments in Programming Under Uncertainty", February, 1990

Invited speaker, Second Asilomar Workshop on Progress in Mathematical Programming, "Progress in Stochastic Programming", February, 1990

Invited speaker, Kieval lectures, Humboldt State University, "The Wide Wide World of Pure Mathematics that Goes by Other Names" and "Solving Linear Programs Under Uncertainty", April, 1990

Tutorial Speaker, TIMS/ORSA Joint National Meeting, Las Vegas, "Developments in Programming Under Uncertainty", May, 1990

Invited speaker, The Eight International Congress of Cybernetics and Systems, Hunter College, "Influences of Decision Science and Computers on Society", June, 1990

Invited speaker, Mathematics Colloquium, The Claremont Colleges, April 1991.
Invited talk, ORSA/TIMS Joint National Meeting, Nashville, May 1991.
Three invited talks, International Symposium on Mathematical Programming, Amsterdam, August 1991.

"Deriving a Utility Function for the U.S. Economy"
"Optimization Under Uncertainty" (SOL 91-4)
"A Polynomially Bounded Algorithm" (SOL 91-5)

*Richard W. Cottle*

Workshop on Generalized Convexity and Fractional Programming, University of California, Riverside, October 10, 1989.
Invited talk, ORSA/TIMS Joint National Meeting, Philadelphia, November 1990.
Invited talk, ORSA/TIMS Joint National Meeting, Nashville, May 1991.
Invited talk, International Symposium on Mathematical Programming, Amsterdam, August 1991.

**HONORS/AWARDS/PRIZES**

George B. Dantzig:

- Gibbs Lecturer, American Mathematical Society, "The Wide Wide World of Pure Mathematics that Goes by Other Names", January , 1990
- First issue of *SIAM Journal of Optimization* dedicated to George Dantzig, February 1991.

Richard W. Cottle:

- Chairman, Department of Operations Research, Stanford University
- Member Faculty Senate, Stanford University
- Chairman, A.W. Tucker Prize Committee, Mathematical Programming Society